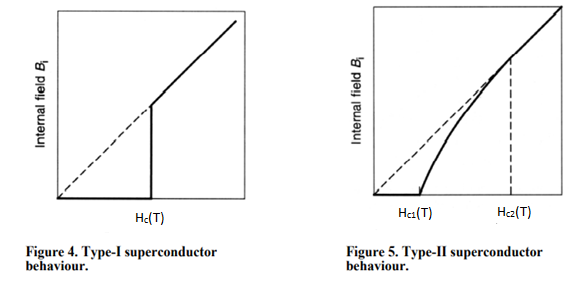
**Meisner Effect (Type II)**

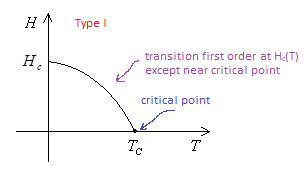
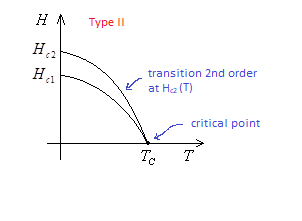
So a superconductor screens out magnetic fields. But as the field is ramped up, it will eventually break the degeneracy between the opposite spin Cooper pairs, making them energetically less favorable to form. And so the superconducting properties of the metal (namely Meisner effect – expulsion of all magnetic field lines, 0 resistance) will eventually vanish. This should start at Tc for even small fields probably. But even at T = 0 for large enough fields. Turns out how it eventually vanishes depends on the type of the superconductor. For Type I’s, at a given T < Tc, raising H = Bf/4π (‘faux’ Gaussian units, for suitable geometry, etc.) up from 0 to Hc(T) doesn’t change the number of Cooper Pairs in the superconductor. But once we cross Hc(T), then all Cooper Pairs dissassociate, the supercurrent dies, and flux is let in. And so apropos Type I’s, our GL Free energy only applies close to Tc, because it’s only there where |ψ|2 is small.

For Type II’s, the number of Cooper Pairs is constant from H = 0 to some critical Hc1(T), and then attenuates until vanishing completely at Hc2(T). Looking at the susceptibility curve below, we can see that even though we have a supercurrent present (because we still have Cooper pairs as asserted) between Hc1(T) and Hc2(T), flux is still allowed to penetrate the material, although less than normally would. Thus, it appears the Meisner effect isn’t realized. The resolution to this paradox seems to be that for type II’s, the Cooper pairs are confined to specific regions of the superconductor after Hc1 is passed so that even though they might carry a super current and can screen out the field within that region, they can’t screen it out outside that region. And this agrees with our conclusion in previous file that if ns were position-dependent, then the Meisner effect wouldn’t happen. The regions where the external field penetrates the sample are called vortices. And their radii goes from 0 at Hc1 to ∞ at Hc2. Hc2 is typically much larger than Hc1. So the transition from superconductor to normal conductor is 2nd order at Hc2. Maybe that’s why they’re called type II superconductors. Well this makes the entire Hc2 border amenable to analysis via the GL free energy since ψ is small.

The susceptibility diagram is:



The basic phase diagrams for type I, II superconductors are:

Just to be clear, we’re saying the order parameter |ψ|2 is small only near Tc for Type I’s, but is small along the entire Hc2(T) curve for type II’s.

**Revisiting Meisner Effect**

So recall we had:



where ψeq(x) is given by:



And recall we derived from this an equation of state,



(b = bound and is same as induced) Let’s say ψeq = √n\*s·eiφ(x), but now we’ll let ns\* be some arbitrary function, not constant. Then let’s work out the current,



and this is still,



So our expression for the current remains the same as when we disallowed ns\* density variations. But as we saw before, the Meisner effect is not realized because ns is presumably spatially dependent now. An interesting consquence of this equation, though, is that whatever flux does penetrate the superconductor must do so in discrete units. Consider the flux through some surface area S,



Now we know that for type I’s, B doesn’t persist past the penetration depth λ.



And since j = ∇×B, neither does j. For type II’s, we haven’t shown this, but I guess B is constant past the penetration depth, and so it’s curl is still zero? And so j still doesn’t persist past the penetration depth (not sure what ns would be in this case – the total Cooper pair number density?) Anyway, that’s what people claim about j, so presuming so, we then have:



where for sake of discussion I’m kind of presuming our contour is going in the x-y (r-θ) plane, perpendicular to the z-axis. Now we might say this expression ought to be 0 for φ continuity reasons. But really, it is *ψ(r,θ,z)* = √nseiφ which must be continuous when making this complete loop around the z-axis. And so all we really need from φ is that Δφ = 2πm, where m is an integer. So then we have:



So whatever flux does penetrate our Type II, must do so in quantized amounts.

**Hc2(T) near critical point for Type II’s**

Analysis is pretty much same. Type II superconductors experience a second order phase transition at Hc2(T), and so the entire border is actually amenable to our analysis. Right? Near here, as we increase H past Hc2(T), the superconductor transitions to a normal conductor, which would mean that the free energy (action) becomes lower with ψ = 0 (and the field consequently penetrating the conductor because no supercurrent and so no screening), than it is with ψ non-zero (i.e. the ψ value which minimizes the free energy action and the field consequently partially – but not totally ‘cause we’ll presume spatially dependent ψ – eliminated from the conductor’s interior because of screening). So we can figure out Hc2(T) by comparing Free energies. Our free energy (really action as of yet) is given by:



and,



But in our scenario, we’re holding not the total field constant, but the free external field constant. So it is the free energy with T and Bf = 4πH (assuming solenoidal geometry, homogeneous substance, and faux Gaussian units) as proper variables, that should be minimized. This free energy, which I’ll call F\* is related to F by a Legendre transformation.



So as we said, the cross-over occurs when F\* is minimized by assuming a normal metal state, in which case ψ = 0 and the external field H penetrates the metal (basically without reduction, as we saw in the free metal file), rather than a superconducting state, in which case ψ(x) assumes the function ψeq(x) which minimizes F\*, although B and A are not totally screened out (will have to effectively work out what they are, and I’m calling them Beq or Aeq). In symbols,



Let’s work on the LHS,



where in the second line we use the fact that the metal is normal and so has negligible response to the external field, and so the total field is just the external field. And in the the third line just presume homogeneity so H should be constant. Now for the RHS,



I’ll do this by finding the spatially varying ψeq which minimizes F, and then work out what Beq is via the relationship cited above, namely that:



So apropos the first objective, we’ll recall from the first file in the folder that when we took the functional derivative of F w/r ψ(x), we found the following equation:



Let’s see if we can make some progress towards solving for F[T,A] allowing for spatially varying ψeq. This equation is probably impossible to solve in generality. I think what we’ll do is use a variational approach. We’ll presume that close to the critical point, where ψ(x) is small, solutions will be similar to what we get when we ignore the u term (ignoring this one because if ψ is small, then ψ4 is very small). And then we’ll take that solution and vary its parameters to minimize the full F, with the u term present. So neglecting u for now, we basically have a Schrodinger-type equation.



So let’s look for solutions to:



We’ll go to the Landau gauge (which is still consistent with the Coulomb gauge) by setting A = (0, Bx, 0). And then we have:



We solved this equation when we analyzed the states of a particle in a magnetic field in the QM folder. We found,



(C is an arbitrary constant which we allow as there is no normalization requirement for ψ here) Now plug this back into our F and hopefully we can minimize F by adjusting these free parameters, ky, kz, n, C. First we’ll make some manipulations on F to facilitate the process:



Plugging in our result will now give us:



Seems clear that at least kz should be zero, in which case En → ωc(n+1/2).



I(n) starts at I(0) = √(π/2) and monotonically decreases from there (not *sure* if it goes to zero, but seems to). It seems that n = 0 gives us the smallest overall result. Assuming so we have:



Now let’s differentiate w/r to C and set to zero,



Remember C2 must be positive here; otherwise solution is just C = 0. Plugging this back in,



So, keeping in mind the stricture in the previous sentence, and filling in ωc = e\*B/m\*, we have:



Now we need to get Beq in terms of H. So using,



We have:



This doesn’t look good. Okay, well I’m just going to say that C2 = -√(2πm\*ωc)(ωc + r)/u, which is the number of Cooper pairs, is small, as it is if we’re close to the phase transition Hc2(T) line. Then in the grossest approximation we can just neglect the entire first term on the RHS, and say,



which makes sense as being correct, close to the transition. So filling B = 4πH into our Fs, we have:



This makes our free energy,



So proceding with our inequality,



and so clearly our result for F\* indicates that our superconductor will be in a metallic state when 4πe\*H/m\* + r > 0, but superconducting otherwise. So Hc2(T) is given by that curve (recall r = 2(T-Tc)/Tc),



A question arises, then. How would we know if a superconductor is going to be Type I vs. Type II? Seems our criterion would just be whether, upon starting off in the metal phase (high H), and dropping H down, keeping T constant (and below Tc), we hit the Hc(T) curve or Hc2(T) curve first. So our superconductor will be Type II, say, if:



We can make this requirement more intelligible if we put it in terms of the two length scales we have (see bottom of the GL Free energy file):



And we have:



So clearly our criterion reduces to (I’m keeping T dependence in arguments of λ and ξ, but can see that their ratio doesn’t depend on T):



So if ξ, the correlation length, is small, the superconductor will want to be Type II. This means that there is relatively little energy cost to allowing the superconducting electron density fluctuate in value. In other words the superconductor isn’t ‘stiff’.

**Abrikosov Lattice**

So we’ll observe that when we were minimizing F\*, we implicitly decided to do so in a restricted eigenspace. The ground state eigenfunctions, ψeq, of our Schrodinger equation were degenerate in kx. But we implicitly chose to work with just the kx = 0 guy. Turns out, that if we’d kept the others, we could’ve found a linear combination of them all which would result in an even lower Free Energy F\*. This linear combination describes how the magnetic field penetrates a Type II superconductor in a lattice of ‘vortices’. But didn’t feel like going into it. Despite our ad hoc neglect of all but one of the degenerate eigenfunctions in our ψeq ansatz, our conclusions apropos the phase separation line Hc2(T) and the Type II vs. Type I criterion, remain valid.